12-7 Graphing Trigonometric Functions

Find the amplitude and period of each function. Then graph the function.

1. $y = 4 \sin \theta$

SOLUTION:

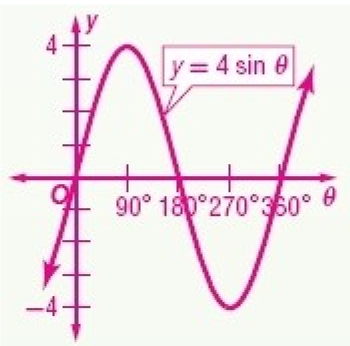
amplitude: $|a| = |4|$ or 4

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$ or 360°

x-intercepts: (0,0)

$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = (180^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (360^\circ, 0)$



2. $y = \sin 3\theta$

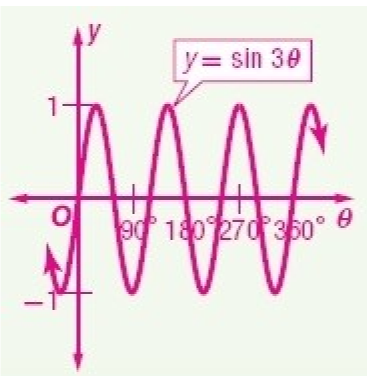
SOLUTION:

amplitude: $|a| = |1|$ or 1

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$ or 120°

x-intercepts: (0,0)

$$\left(\frac{1}{2}\cdot\frac{360^\circ}{b}, 0\right) = (60^\circ, 0)$$
$$\left(\frac{360^\circ}{b}, 0\right) = (120^\circ, 0)$$



12-7 Graphing Trigonometric Functions

3. $y = \cos 2\theta$

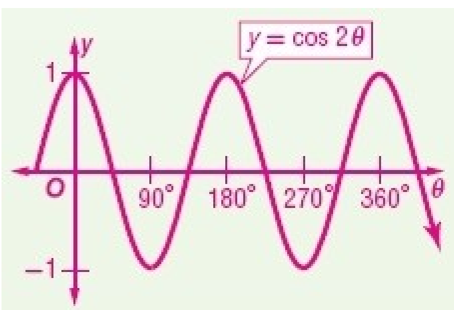
SOLUTION:

amplitude: $|a| = |1|$ or 1

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|2|}$ or 180°

x-intercepts:

$$\left(\frac{1}{4} \cdot \frac{360^\circ}{b}, 0\right) = (45^\circ, 0)$$
$$\left(\frac{3}{4} \cdot \frac{360^\circ}{b}, 0\right) = (135^\circ, 0)$$



4. $y = \frac{1}{2}\cos 3\theta$

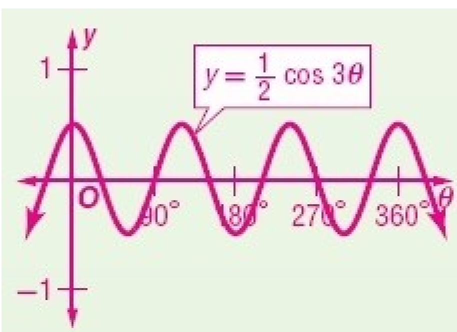
SOLUTION:

amplitude: $|a| = \left|\frac{1}{2}\right|$ or $\frac{1}{2}$

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$ or 120°

x-intercepts:

$$\left(\frac{1}{4}\cdot\frac{360^\circ}{b}, 0\right) = (30^\circ, 0)$$
$$\left(\frac{3}{4}\cdot\frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$$



5. **SPIDERS** When an insect gets caught in a spider web, the web vibrates with a frequency of 14 hertz.

a. Find the period of the function.

b. Let the amplitude equal 1 unit. Write a sine equation to represent the vibration of the web y as a function of time *t*. Then graph the equation.

SOLUTION:

a. The period of the function is $\frac{1}{14}$ or about 0.07.

b. Period = $\frac{2\pi}{|b|}$

$$\frac{1}{14} = \frac{2\pi}{|b|}$$
$$\frac{1}{14}|b| = 2\pi$$
$$b = 28\pi$$

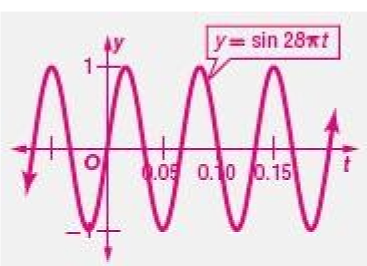
Substitute 1 for a, 28π for b and t for θ in the general equation for the sine function.

$$y = a \sin b\theta$$
$$y = 1 \sin 28\pi t$$
$$y = \sin 28\pi t$$

amplitude: $|a| = |1|$ or 1

x-intercepts: (0, 0)

$$\left(\frac{1}{2} \cdot \frac{2\pi}{b}, 0\right) = (0.036, 0)$$
$$\left(\frac{2\pi}{b}, 0\right) = (0.07, 0)$$



12-7 Graphing Trigonometric Functions

Find the period of each function. Then graph the function.

6. $y = 3 \tan \theta$

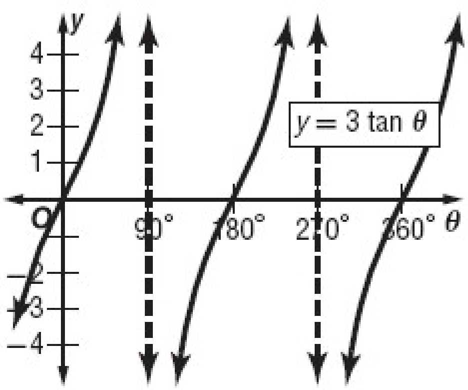
SOLUTION:

period: $\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$ or 180°

asymptotes: $\frac{180^\circ}{2|b|} = \frac{180^\circ}{2|1|}$ or 90°

Sketch asymptotes at $1 \cdot 90^\circ$ or 90°, $3 \cdot 90^\circ$ or 270°, and so on.

Use $y = \tan \theta$, draw one cycle every 180º.



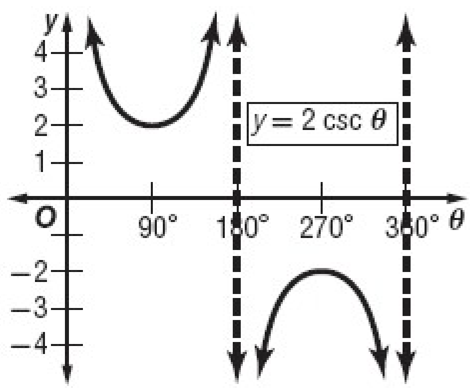
7. $y = 2 \csc \theta$

SOLUTION:

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$ or 360°

Since 2 csc θ is a reciprocal of 2 sin θ, the vertical asymptotes occur at the points where 2 sin θ = 0. So, the asymptotes are at $\theta = 0°$ and $\theta = 180°$.

Sketch $y = 2 \csc \theta$.

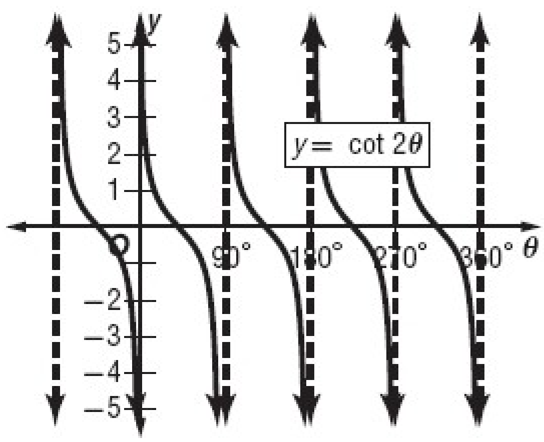


8. $y = \cot 2\theta$

SOLUTION:

period: $\frac{180^\circ}{|b|} = \frac{180^\circ}{|2|}$ or 90°

Since $\cot 2\theta$ is a reciprocal of $\tan 2\theta$, the vertical asymptotes occur at the points where $\tan 2\theta = 0$. So, the asymptotes are at $\theta = 0°$, $\theta = 90°$, $\theta = 180°$, $\theta = 270°$ and so on
.
Sketch $y = \cot 2\theta$.



12-7 Graphing Trigonometric Functions

Find the amplitude and period of each function. Then graph the function.

9. $y = 2 \cos \theta$

SOLUTION:

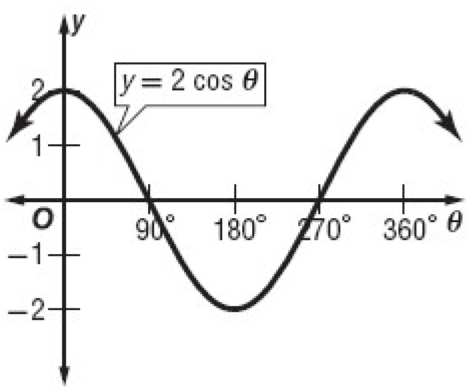
amplitude: $|a| = |2|$ or 2

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$ or 360°

x-intercepts:

$$\left(\frac{1}{4} \cdot \frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$$

$$\left(\frac{3}{4} \cdot \frac{360^\circ}{b}, 0\right) = (270^\circ, 0)$$



10. $y = 3 \sin \theta$

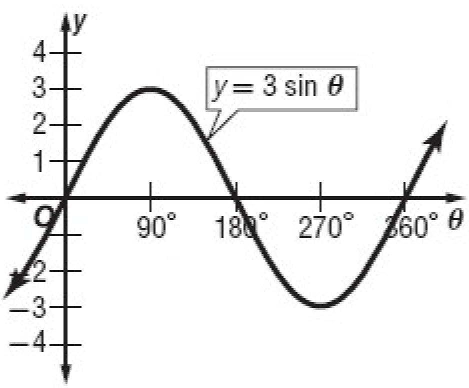
SOLUTION:

amplitude: $|a| = |3|$ or 3

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$ or 360°

x-intercepts: (0,0)

$$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = (180^\circ, 0)$$
$$\left(\frac{360^\circ}{b}, 0\right) = (360^\circ, 0)$$



11. $y = \sin 2\theta$

SOLUTION:

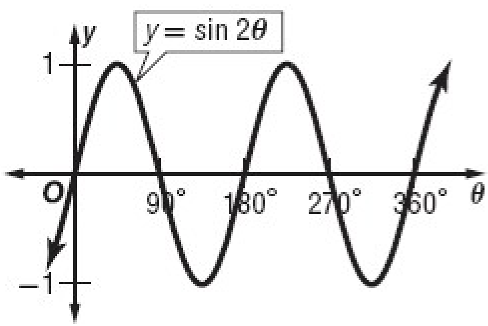
amplitude: $|a| = |1|$ or 1

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|2|}$ or 180°

x-intercepts: (0,0)

$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (180^\circ, 0)$



12-7 Graphing Trigonometric Functions

12. $y = \cos 3\theta$

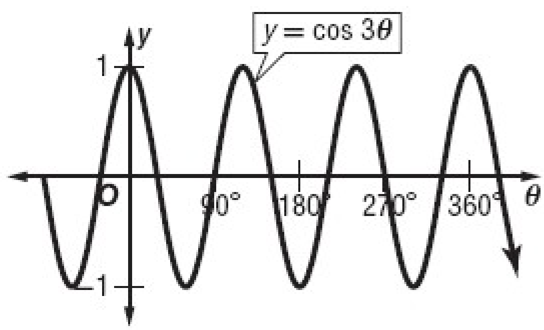
SOLUTION:

amplitude: $|a| = |1|$ or 1

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$ or 120°

x-intercepts:

$$\left(\frac{1}{4} \cdot \frac{360^\circ}{b}, 0\right) = (30^\circ, 0)$$
$$\left(\frac{3}{4} \cdot \frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$$



13. $y = \cos \frac{1}{2}\theta$

SOLUTION:

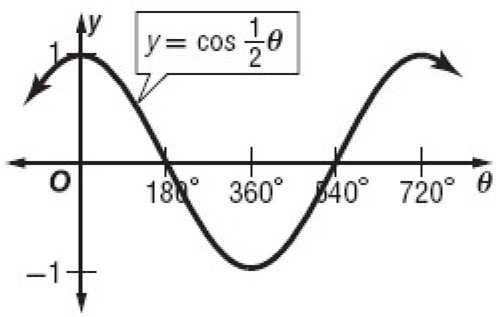
amplitude: $|a| = |1|$ or 1

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{\left|\frac{1}{2}\right|}$ or 720°

x-intercepts:

$\left(\frac{1}{4} \cdot \frac{360^\circ}{b}, 0\right) = (180^\circ, 0)$

$\left(\frac{3}{4} \cdot \frac{360^\circ}{b}, 0\right) = (540^\circ, 0)$



14. $y = \sin 4\theta$

SOLUTION:

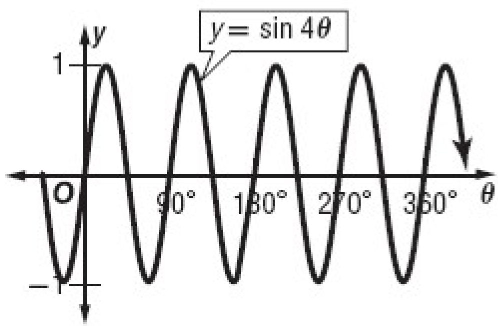
amplitude: $|a| = |1|$ or 1

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|4|}$ or 90°

x-intercepts: (0,0)

$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = (45^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$



15. $y = \frac{3}{4}\cos\theta$

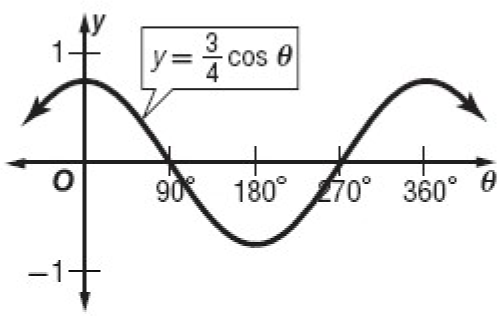
SOLUTION:

amplitude: $|a| = \left|\frac{3}{4}\right|$ or $\frac{3}{4}$

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$ or 360°

x-intercepts:

$$\left(\frac{1}{4}\cdot\frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$$
$$\left(\frac{3}{4}\cdot\frac{360^\circ}{b}, 0\right) = (270^\circ, 0)$$



16. $y = \frac{3}{2}\sin\theta$

SOLUTION:

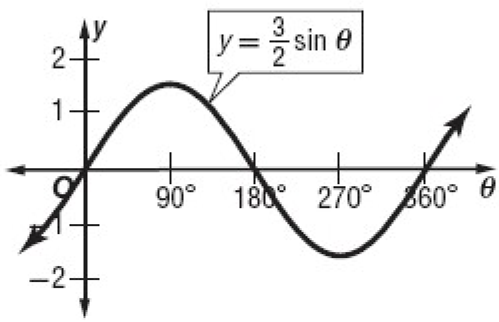
amplitude: $|a| = \left|\frac{3}{2}\right|$ or $\frac{3}{2}$

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$ or 360°

x-intercepts: (0,0)

$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = (180^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (360^\circ, 0)$



17. $y = \frac{1}{2}\sin 2\theta$

SOLUTION:

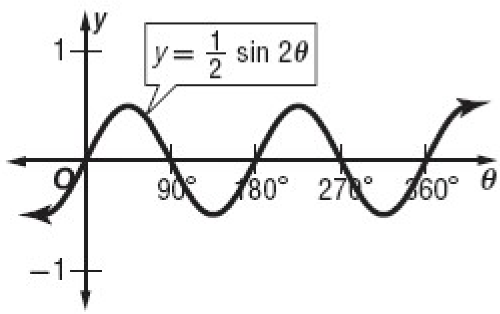
amplitude: $|a| = \left|\frac{1}{2}\right|$ or $\frac{1}{2}$

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|2|}$ or 180°

x-intercepts: (0,0)

$\left(\frac{1}{2}\cdot\frac{360^\circ}{b}, 0\right) = (90^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (180^\circ, 0)$



18. $y = 4 \cos 2\theta$

SOLUTION:

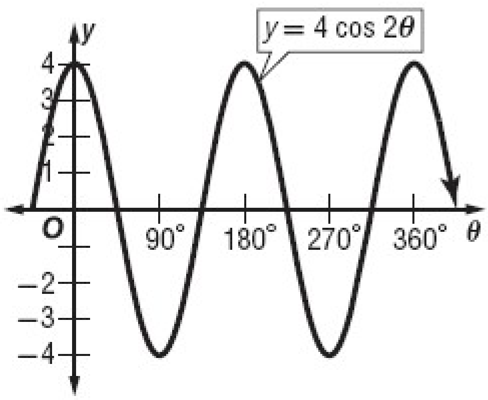
amplitude: $|a| = |4|$ or 4

period: $\frac{360°}{|b|} = \frac{360°}{|2|}$ or 180°

x-intercepts:

$\left(\frac{1}{4} \cdot \frac{360°}{b}, 0\right) = (45°, 0)$

$\left(\frac{3}{4} \cdot \frac{360°}{b}, 0\right) = (135°, 0)$



19. $y = 3 \cos 2\theta$

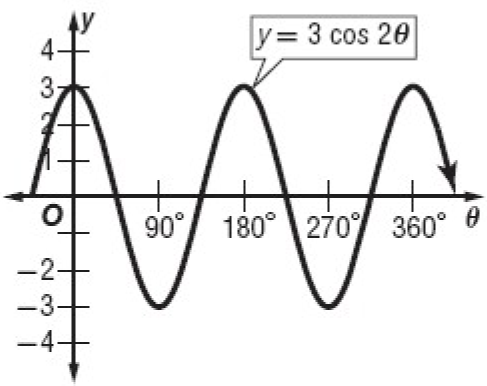
SOLUTION:

amplitude: $|a| = |3|$ or 3

period: $\frac{360°}{|b|} = \frac{360°}{|2|}$ or $180°$

x-intercepts:

$\left(\frac{1}{4} \cdot \frac{360°}{b}, 0\right) = (45°, 0)$
$\left(\frac{3}{4} \cdot \frac{360°}{b}, 0\right) = (135°, 0)$



20. $y = 5\sin\frac{2}{3}\theta$

SOLUTION:

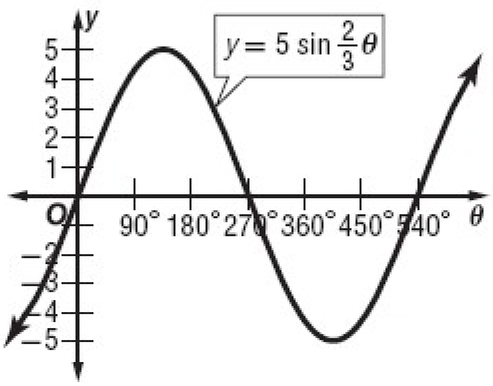
amplitude: $|a| = |5|$ or 5

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{\left|\frac{2}{3}\right|}$ or 540°

x-intercepts: (0,0)

$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = (270^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (540^\circ, 0)$



21. **CCSS REASONING** A boat on a lake bobs up and down with the waves. The difference between the lowest and highest points of the boat is 8 inches. The boat is at *equilibrium* when it is halfway between the lowest and highest points. Each cycle of the periodic motion lasts 3 seconds.

a. Write an equation for the motion of the boat. Let *h* represent the height in inches and let *t* represent the time in seconds. Assume that the boat is at equilibrium at *t* = 0 seconds.

b. Draw a graph showing the height of the boat as a function of time.

SOLUTION:

a. The period of the function is 3.

Period = $\frac{2\pi}{|b|}$

$$3 = \frac{2\pi}{|b|}$$
$$3|b| = 2\pi$$
$$b = \frac{2\pi}{3}$$

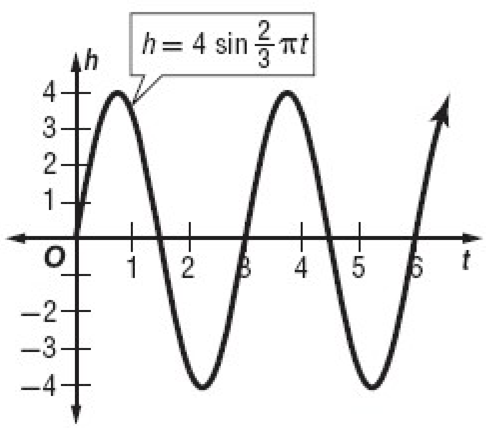
Substitute 4 for *a*, $\frac{2\pi}{3}$ for *b* and *t* for *θ* in the general equation for the sine function.

$$y = a \sin b\theta$$
$$y = 4 \sin \frac{2}{3}\pi t$$

b.
amplitude: $|a| = |4|$ or 4

x-intercepts: (0,0)

$$\left(\frac{1}{2} \cdot \frac{2\pi}{b}, 0\right) = (1.5, 0)$$
$$\left(\frac{2\pi}{b}, 0\right) = (3, 0)$$



22. **ELECTRICITY** The voltage supplied by an electrical outlet is a periodic function that *oscillates*, or goes up and down, between –165 volts and 165 volts with a frequency of 50 cycles per second.

a. Write an equation for the voltage *V* as a function of time *t*. Assume that at *t* = 0 seconds, the current is 165 volts.

b. Graph the function.

SOLUTION:

a. The period of the function per second is $\frac{1}{50}$ or 0.02.

$$\text{Period} = \frac{2\pi}{|b|}$$
$$0.02 = \frac{2\pi}{|b|}$$
$$0.02|b| = 2\pi$$
$$b = \frac{2\pi}{0.02}$$
$$b = 100\pi$$

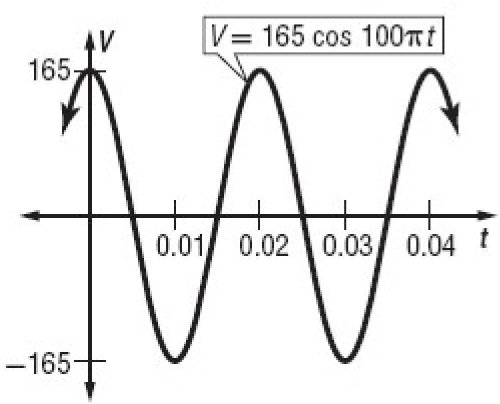
Substitute 165 for *a*, 100π for *b* and *t* for *θ* in the general equation for the cosine function.

$$y = a\cos b\theta$$
$$y = 165\cos 100\pi t$$

b. amplitude: $|a| = |165|$ or 165

x-intercepts:

$$\left(\frac{1}{4}\cdot\frac{2\pi}{b}, 0\right) = (0.005, 0)$$
$$\left(\frac{3}{4}\cdot\frac{2\pi}{b}, 0\right) = (0.015, 0)$$



Find the period of each function. Then graph the function.

23. $y = \tan \frac{1}{2}\theta$

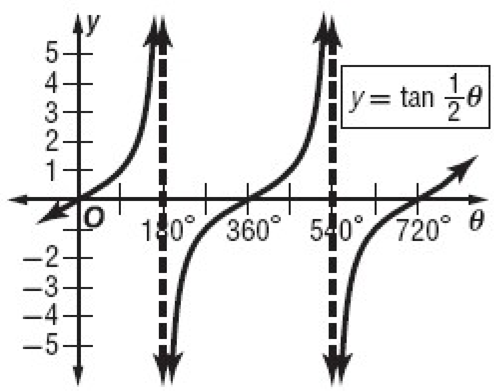
SOLUTION:

period: $\frac{180^\circ}{|b|} = \frac{180^\circ}{\left|\frac{1}{2}\right|}$ or 360°

asymptotes: $\frac{180^\circ}{2|b|} = \frac{180^\circ}{2\left|\frac{1}{2}\right|}$ or 180°

Sketch asymptotes at $-1 \cdot 180^\circ$ or $-180^\circ, 1 \cdot 180^\circ$ or $180^\circ, 3 \cdot 180^\circ$ or 540° and so on.

Use $y = \tan \theta$, draw one cycle every 360º.



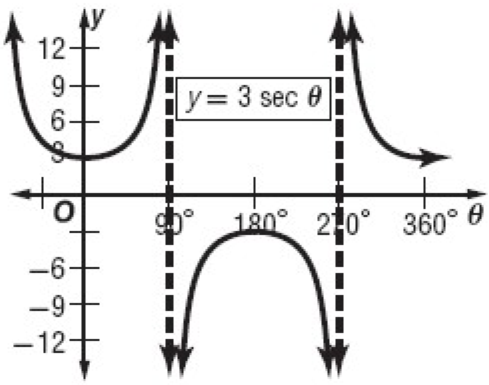
24. $y = 3 \sec \theta$

SOLUTION:

Period of the function is 360º.

Since 3 sec θ is a reciprocal of 3 cos θ, the vertical asymptotes occur at the points where 3 cos θ = 0. So, the asymptotes are at θ = 90° and θ = 270°.

Sketch $y = 3 \cos \theta$ and use it to graph $y = 3 \sec \theta$.



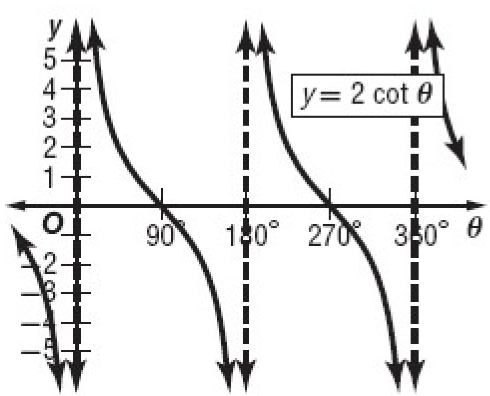
25. $y = 2 \cot \theta$

SOLUTION:

period: $\frac{180}{|b|} = \frac{180}{|1|}$ or 180

Since 2 cot θ is a reciprocal of 2 tan θ, the vertical asymptotes occur at the points where 2 tan θ = 0. So, the asymptotes are at θ = 0°, θ = 180°, θ = 360° and so on.

Sketch $y = 2 \tan \theta$ and use it to graph $y = 2 \cot \theta$.



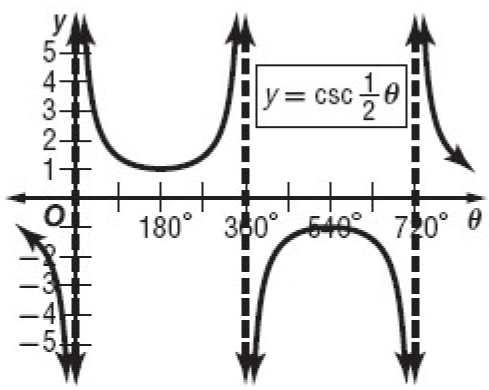
26. $y = \csc \frac{1}{2}\theta$

SOLUTION:

period: $\frac{360}{|b|} = \frac{360}{\left|\frac{1}{2}\right|}$ or 720

Since $\csc \frac{1}{2}\theta$ is a reciprocal of $\sin \frac{1}{2}\theta$, the vertical asymptotes occur at the points where $\sin \frac{1}{2}\theta = 0$. So, the asymptotes are at $\theta = 0°$, $\theta = 360°$, $\theta = 720°$ and so on.

Sketch $y = \sin \frac{1}{2}\theta$ and use it to graph $y = \csc \frac{1}{2}\theta$.



27. $y = 2\tan\theta$

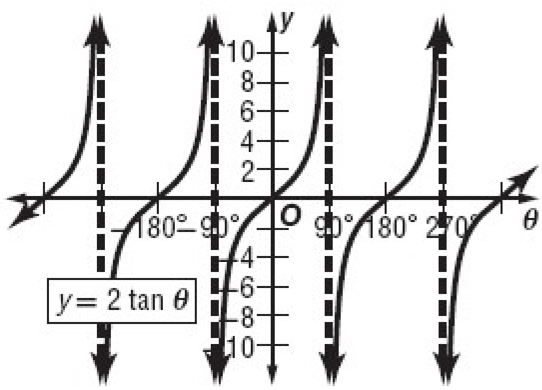
SOLUTION:

period: $\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$ or 180°

asymptotes: $\frac{180^\circ}{2|b|} = \frac{180^\circ}{2|1|}$ or 90°

Sketch asymptotes at $-1 \cdot 90^\circ$ or -90°, $1 \cdot 90^\circ$ or 90°, $3 \cdot 90^\circ$ or 270°, and so on.

Use $y = \tan\theta$, draw one cycle every 180º.



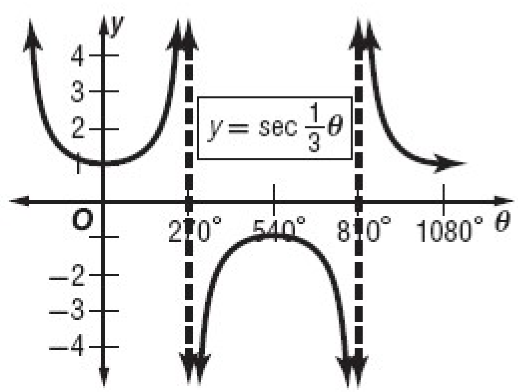
28. $y = \sec \frac{1}{3}\theta$

SOLUTION:

period: $\frac{360}{|b|} = \frac{360}{\left|\frac{1}{3}\right|}$ or 1080

Since $\sec \frac{1}{3}\theta$ is a reciprocal of $\cos \frac{1}{3}\theta$, the vertical asymptotes occur at the points where $\cos \frac{1}{3}\theta = 0$. So, the asymptotes are at $\theta = -270°$, $\theta = 270°$, $\theta = 810°$ and so on.

Sketch $y = \cos \frac{1}{3}\theta$ and use it to graph $y = \sec \frac{1}{3}\theta$.



29. **EARTHQUAKES** A seismic station detects an earthquake wave that has a frequency of 0.5 hertz and an amplitude of 1 meter.

a. Write an equation involving sine to represent the height of the wave *h* as a function of time *t*. Assume that the equilibrium point of the wave, $h = 0$, is halfway between the lowest and highest points.

b. Graph the function. Then determine the height of the wave after 20.5 seconds.

SOLUTION:

a. The period of the function is $\frac{1}{0.5}$ or 2.

Period = $\frac{2\pi}{|b|}$

$$\begin{aligned} 2 &= \frac{2\pi}{|b|} \\ 2|b| &= 2\pi \\ b &= \pi \end{aligned}$$

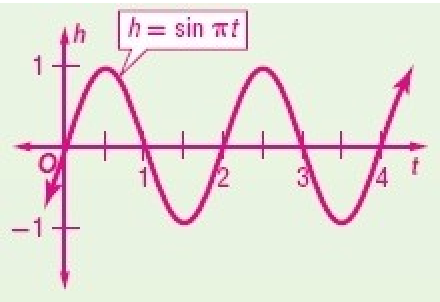
Substitute 1 for *a*, π for *b* and *t* for *θ* in the general equation for the sine function.

$$\begin{aligned} y &= a\sin b\theta \\ y &= 1\sin \pi t \\ y &= \sin \pi t \end{aligned}$$

b. amplitude: $|a| = |1|$ or 1

x-intercepts: (0,0)

$$\left(\frac{1}{2}\cdot\frac{2\pi}{b}, 0\right) = (1, 0)$$
$$\left(\frac{2\pi}{b}, 0\right) = (2, 0)$$



30. **CCSS PERSEVERANCE** An object is attached to a spring as shown at the right. It oscillates according to the equation $y = 20 \cos \pi t$, where y is the distance in centimeters from its equilibrium position at time *t*.

a. Describe the motion of the object by finding the following: the amplitude in centimeters, the frequency in vibrations per second, and the period in seconds.

b. Find the distance of the object from its equilibrium position at $t = \frac{1}{4}$ second.

c. The equation $v = (-20 \text{ cm})(\pi \text{ rad/s}) \cdot \sin(\pi \text{ rad/s} \cdot t)$ represents the velocity v of the object at time t. Find the velocity at $t = \frac{1}{4}$ second.

SOLUTION:

a. amplitude: $|a| = |20|$ or 20 cm

period: $\frac{2\pi}{|b|} = \frac{2\pi}{|\pi|}$ or 2 seconds

Frequency is the reciprocal of period.

So, the frequency is $\frac{1}{2}$ or 0.5 vibrations per second.

b. Substitute $t = \frac{1}{4}$ in the given equation and solve for y.

$$y = 20\cos \pi t$$
$$y = 20\cos \pi\left(\frac{1}{4}\right)$$
$$y = 20\cos 45$$
$$y \approx 14.1$$

The distance of the object from the equilibrium position is about 14.1 cm at $t = \frac{1}{4}$ second.

c. Substitute $t = \frac{1}{4}$ in the given equation and solve for v.

$$v = -20\pi \sin \pi t$$

$$\begin{aligned} v &= -20\pi \sin \frac{\pi}{4} \\ &= -20\pi\left(\frac{1}{\sqrt{2}}\right) \\ &= -10\pi\sqrt{2} \\ &\approx -44.4 \end{aligned}$$

The velocity is about –44.4 cm/s.

31. **PIANOS** A piano string vibrates at a frequency of 130 hertz.

a. Write and graph an equation using cosine to model the vibration of the string y as a function of time *t*. Let the amplitude equal 1 unit.

b. Suppose the frequency of the vibration doubles. Do the amplitude and period increase, decrease, or remain the same? Explain.

SOLUTION:

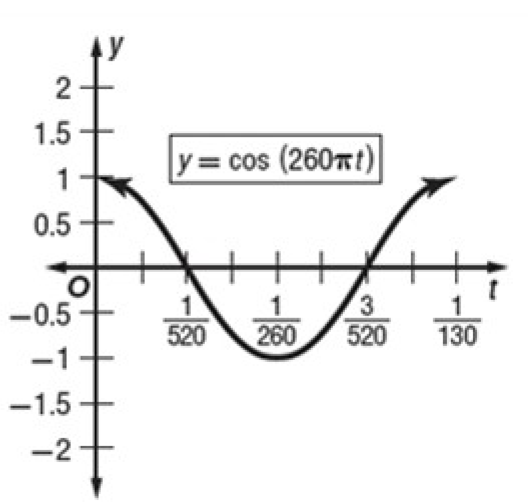
a. The period of the function is $\frac{1}{130}$.

$$\text{Period} = \frac{2\pi}{|b|}$$
$$\frac{1}{130} = \frac{2\pi}{|b|}$$
$$\frac{1}{130}|b| = 2\pi$$
$$b = 260\pi$$

Substitute 1 for a, 260π for b and t for θ in the general equation for the cosine function.

$$y = a\cos b\theta$$
$$y = 1\cos 260\pi t$$
$$y = \cos 260\pi t$$

Graph the function.



b. The amplitude remains the same. The period decreases because it is the reciprocal of the frequency.

Find the amplitude, if it exists, and period of each function. Then graph the function.

32. $y = 3\sin\frac{2}{3}\theta$

SOLUTION:

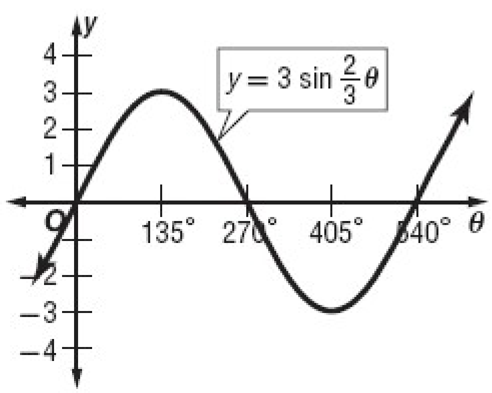
amplitude: $|a| = |3|$ or 3

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{\left|\frac{2}{3}\right|}$ or 540°

x-intercepts: (0,0)

$\left(\frac{1}{2}\cdot\frac{360^\circ}{b}, 0\right) = (270^\circ, 0)$

$\left(\frac{360^\circ}{b}, 0\right) = (540^\circ, 0)$



33. $y = \frac{1}{2}\cos\frac{3}{4}\theta$

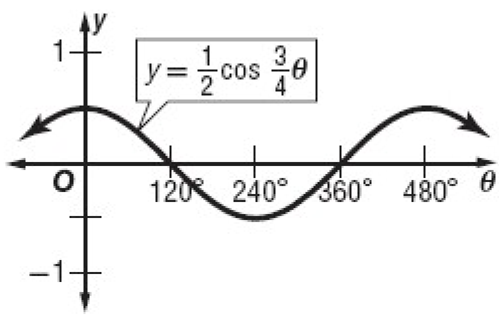
SOLUTION:

amplitude: $|a| = \left|\frac{1}{2}\right|$ or $\frac{1}{2}$

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{\left|\frac{3}{4}\right|}$ or 480°

x-intercepts:

$$\left(\frac{1}{4}\cdot\frac{360^\circ}{b}, 0\right) = (120^\circ, 0)$$
$$\left(\frac{3}{4}\cdot\frac{360^\circ}{b}, 0\right) = (360^\circ, 0)$$



34. $y = 2\tan\frac{1}{2}\theta$

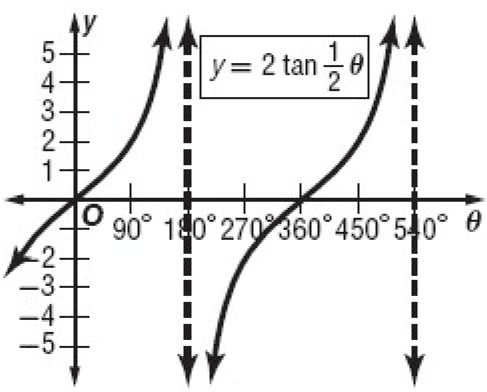
SOLUTION:
amplitude: does not exist

period: $\frac{180^\circ}{|b|} = \frac{180^\circ}{\left|\frac{1}{2}\right|}$ or 360°

asymptotes: $\frac{180^\circ}{2|b|} = \frac{180^\circ}{2\left|\frac{1}{2}\right|}$ or 180°

Sketch asymptotes at $-1 \cdot 180^\circ$ or -180°, $1 \cdot 180^\circ$ or 180°, $3 \cdot 180^\circ$ or 540°, and so on.

Use $y = \tan\theta$, draw one cycle every 360º.



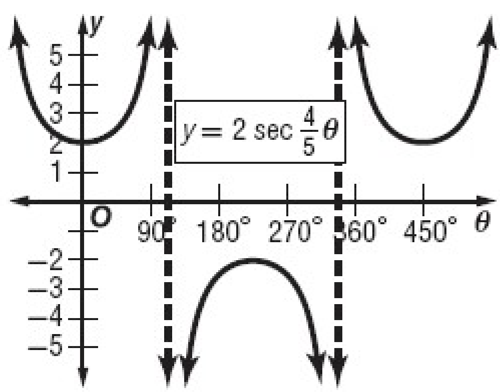
35. $y = 2\sec\frac{4}{5}\theta$

SOLUTION:
amplitude: does not exist.

period: $\frac{360}{|b|} = \frac{360}{\left|\frac{4}{5}\right|}$ or 450

The vertical asymptotes occur at the points where 2 cos $\frac{4}{5}\theta$ = 0. So, the asymptotes are at θ = 112.5º and θ = 337.5º.

Sketch 2 cos $\frac{4}{5}\theta$ and use it to graph 2 sec $\frac{4}{5}\theta$.



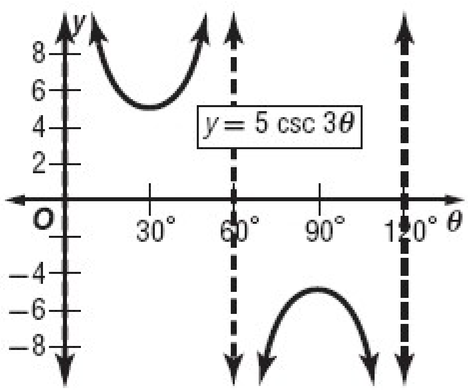
36. $y = 5 \csc 3\theta$

SOLUTION:
amplitude: does not exist.

period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|}$ or 120°

The vertical asymptotes occur at the points where $5 \sin 3\theta = 0$. So, the asymptotes are at $\theta = 60º$ and $\theta = 120º$.

Sketch $y = 5 \sin 3\theta$ and use it to graph $y = 5 \csc 3\theta$.



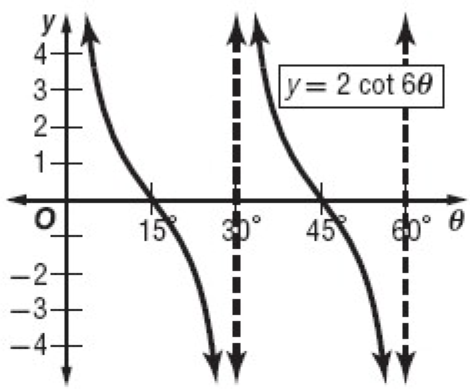
37. $y = 2 \cot 6\theta$

SOLUTION:
amplitude: does not exist

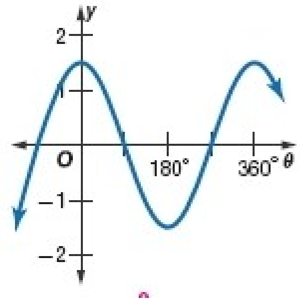
period: $\frac{180°}{|b|} = \frac{180°}{|6|}$ or $30°$

The vertical asymptotes occur at the points where $2 \tan 6\theta = 0$. So, the asymptotes are at $\theta = 0°$, $\theta = 30°$, $\theta = 90°$ and so on.

Sketch $y = 2 \tan 6\theta$ and use it to graph $y = 2 \cot 6\theta$.



Identify the period of the graph and write an equation for each function.

38. 

SOLUTION:

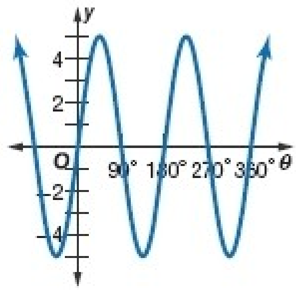
The period of the graph is 360º.

The amplitude of the graph is $\frac{3}{2}$.

$$\text{Period} = \frac{360^\circ}{|b|}$$
$$360^\circ = \frac{360^\circ}{|b|}$$
$$360^\circ|b| = 360^\circ$$
$$b = 1$$

Substitute $\frac{3}{2}$ for a, 1 for b in the general equation for the cosine function.

$$y = a\cos b\theta$$
$$y = \frac{3}{2}\cos(1)\theta$$
$$y = \frac{3}{2}\cos\theta$$

39. 

SOLUTION:

The period of the graph is 180º.
The amplitude of the graph is 5.

$$\text{Period} = \frac{360^\circ}{|b|}$$

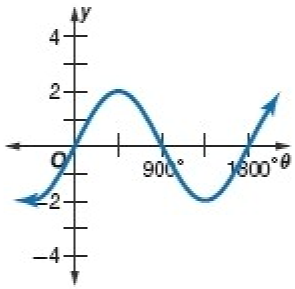
$$180^\circ = \frac{360^\circ}{|b|}$$

$$180^\circ |b| = 360^\circ$$

$$b = 2$$

Substitute 5 for a, 2 for b in the general equation for the sine function.

$$y = a \sin b\theta$$
$$y = 5 \sin 2\theta$$

40. 

SOLUTION:
The period of the graph is 1800º.

The amplitude of the graph is 2.

$$\text{Period} = \frac{360}{|b|}$$
$$1800 = \frac{360}{|b|}$$
$$|b| = \frac{360}{1800}$$
$$b = \frac{1}{5}$$

Substitute 2 for a, $\frac{1}{5}$ for b in the general equation for the sine function.

$$y = a\sin b\theta$$
$$y = 2\sin\frac{1}{5}\theta$$

41. **CHALLENGE** Describe the domain and range of $y = a \cos \theta$ and $y = a \sec \theta$, where a is any positive real number.

SOLUTION:
The domain of $y = a \cos \theta$ is the set of all real numbers.
The domain of $y = a \sec \theta$ is the set of all real numbers except the values for which $\cos \theta = 0$.
The range of $y = a \cos \theta$ is $-a \le y \le a$.
The range of $y = a \sec \theta$ is $y \le -a$ and $y \ge a$.

42. **REASONING** Compare and contrast the graphs of $y=\frac{1}{2}\sin\theta$ and $y=\sin\frac{1}{2}\theta$.

SOLUTION:

The graph of $y=\frac{1}{2}\sin\theta$ has an amplitude of $\frac{1}{2}$ and a period of 360°.

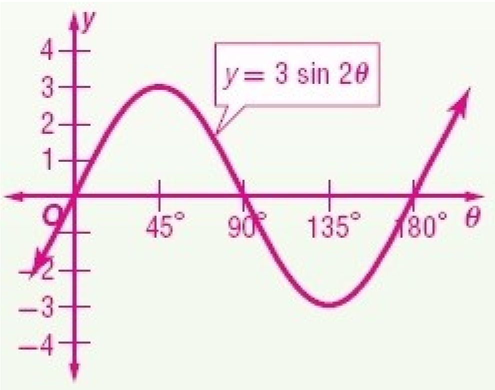
The graph of $y=\sin\frac{1}{2}\theta$ has an amplitude of 1 and a period of 720°.

43. **OPEN ENDED** Write a trigonometric function that has an amplitude of 3 and a period of 180°. Then graph the function.

SOLUTION:

Sample answer:

$y = 3\sin 2\theta$



44. **WRITING IN MATH** How can you use the characteristics of a trigonometric function to sketch its graph?

SOLUTION:

Sample answer: Determine the amplitude and period of the function; find and graph any x-intercepts, extrema, and asymptotes; use the parent function to sketch the graph.

45. **SHORT RESPONSE** Find the 100,001st term of the sequence.

13, 20, 27, 34, 41, …

SOLUTION:

Substitute a_1 = 13 and d = 7 in the formula to find the nth term.

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_{100,001} &= 13 + (100,001 - 1)7 \\ &= 13 + (100,000)7 \\ &= 13 + 700,000 \\ &= 700,013 \end{aligned}$$

46. **STATISTICS** You bowled five games and had the following scores: 143, 171, 167, 133, and 156. What was your average?

A 147

B 153

C 154

D 156

SOLUTION:

Average = $\frac{143+171+167+133+156}{5}$ or 154

C is the correct option.

47. Your city had a population of 312,430 ten years ago. If its current population is 418,270, by what percentage has it grown over the past 10 years?

F 25%

G 34%

H 66%

J 75%

SOLUTION:

Percentage of growth over the past 10 years

$$\begin{aligned}
&= \frac{\text{Change in population}}{\text{Original population}} \times 100\% \\
&= \frac{418,270 - 312,430}{312,430} \times 100\% \\
&= \frac{105,840}{312,430} \times 100\% \\
&\approx 34\%
\end{aligned}$$

G is the correct option.

48. **SAT/ACT** If $h + 4 = b - 3$, then $(h - 2)^2 =$

A $h^2 + 4$

B $b^2 - 6b + 3$

C $b^2 - 18b + 81$

D $b^2 - 14b + 49$

E $b^2 - 10b + 25$

SOLUTION:

$$\begin{aligned} h + 4 &= b - 3 \\ h &= b - 7 \end{aligned}$$

Substitute $b - 7$ for h.

$$\begin{aligned} (h-2)^2 &= (b-7-2)^2 \\ &= (b-9)^2 \\ &= b^2 - 18b + 81 \end{aligned}$$

C is the correct option.

Find the exact value of each expression.

49. $\cos 120° - \sin 30°$

SOLUTION:

$$\begin{aligned} \cos 120° - \sin 30° &= -\frac{1}{2} - \frac{1}{2} \\ &= -1 \end{aligned}$$

50. 3(sin 45°)(sin 60°)

SOLUTION:

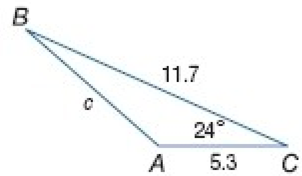
$$\begin{aligned}3(\sin 45^\circ)(\sin 60^\circ) &= 3\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{3\sqrt{6}}{4}\end{aligned}$$

51. $4\sin\frac{4\pi}{3} - 2\cos\frac{\pi}{6}$

SOLUTION:

$$\begin{aligned}4\sin\frac{4\pi}{3} - 2\cos\frac{\pi}{6} &= 4\left(-\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\\ &= -2\sqrt{3} - \sqrt{3}\\ &= -3\sqrt{3}\end{aligned}$$

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

52. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

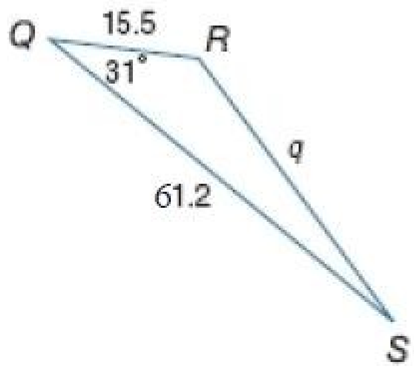
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 11.7^2 + 5.3^2 - 2(11.7)(5.3)\cos 24^\circ$$
$$c \approx 7.2$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin B}{5.3} \approx \frac{\sin 24^\circ}{7.2}$$
$$\sin B \approx \frac{5.3\sin 24^\circ}{7.2}$$
$$B \approx 17^\circ$$

Find the measure of $\angle A$.

$$\angle A \approx 180^\circ - (17^\circ + 24^\circ) \text{ or } 139^\circ$$

53. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

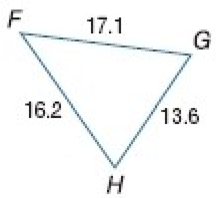
$$q^2 = r^2 + s^2 - 2rs\cos Q$$
$$q^2 = 61.2^2 + 15.5^2 - 2(61.2)(15.5)\cos 31^\circ$$
$$q \approx 48.6$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin R}{61.2} \approx \frac{\sin 31^\circ}{48.6}$$
$$\sin R \approx \frac{61.2\sin 31^\circ}{48.6}$$
$$R \approx 40.4^\circ$$
$$R = 180^\circ - 40^\circ = 140^\circ (\text{since R is an obtuse angle})$$

Find the measure of $\angle S$

$$\angle S \approx 180^\circ - (140^\circ + 31^\circ)$$
$$\approx 9^\circ$$

54. 

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle $\angle H$.

$$\begin{aligned} h^2 &= f^2 + g^2 - 2fg\cos H \\ 17.1^2 &= 13.6^2 + 16.2^2 - 2(13.6)(16.2)\cos H \\ \frac{17.1^2 - 13.6^2 - 16.2^2}{-2(13.6)(16.2)} &= \cos H \\ 69^\circ &\approx H \end{aligned}$$

Use the Law of Sines to find the measure of angle, $\angle G$.

$$\begin{aligned} \frac{\sin G}{16.2} &\approx \frac{\sin 69^\circ}{17.1} \\ \sin G &\approx \frac{16.2\sin 69^\circ}{17.1} \\ G &\approx 62^\circ \end{aligned}$$

Find the measure of $\angle F$.

$$\angle F \approx 180^\circ - (69^\circ + 62^\circ) \text{ or } 49^\circ$$

A binomial distribution has a 40% rate of success. There are 12 trials.

55. What is the probability that there will be exactly 5 failures?

SOLUTION:

The probability of a success is 0.4.

The probability of a failure is 1 – 0.4 or 0.6.

The probability of 3 failures is ${}_{12}C_5(0.6)^5(0.4)^7$or about 0.101, or 10.1%.

56. What is the probability that there will be at least 8 successes?

SOLUTION:

The probability that at least 8 successes is
$_{12}C_8(0.4)^8(0.6)^4 + {}_{12}C_9(0.4)^9(0.6)^3 + \ldots + {}_{12}C_{12}(0.4)^{12}(0.6)^0$.

That is approximately 0.057, or 5.7%.

57. What is the expected number of successes?

SOLUTION:

Expected value of binomial distribution.

$$\begin{aligned} E(X) &= np \\ &= 12(0.4) \\ &= 4.8 \end{aligned}$$

The expected number of success is a bout 5.

58. **BANKING** Rita has deposited \$1000 in a bank account. At the end of each year, the bank posts interest to her account in the amount of 3% of the balance, but then takes out a \$10 annual fee.

a. Let b_0 be the amount Rita deposited. Write a recursive equation for the balance b_n in her account at the end of n years.

b. Find the balance in the account after four years.

SOLUTION:

a. The recursive equation for the balance b_n at the end of n years is

$$b_n = \left(b_{n-1} + \left(b_{n-1} \cdot 0.03\right)\right) - 10$$
$$b_n = b_{n-1} + 0.03b_{n-1} - 10$$
$$b_n = 1.03b_{n-1} - 10$$

b.

$$b_0 = \$1000$$
$$b_1 = 1.03(1000) - 10 = \$1020$$
$$b_2 = 1.03(1020) - 10 = \$1040.6$$
$$b_3 = 1.03(1040.6) - 10 = \$1061.818$$
$$b_4 = 1.03(1061.818) - 10 \approx \$1083.67$$

After four years Rita will have \$1083.67 in her account.

Write an equation for an ellipse that satisfies each set of conditions.

59. center at (6, 3), focus at (2, 3), co-vertex at (6, 1)

SOLUTION:

Since the y-coordinate of the center and the focus are same, the orientation is horizontal.
From the given points: $h = 6$, $k = 3$, $h - c = 2$, $k - b = 1$.

Find the values of c.

$$6 - c = 2$$
$$c = 4$$

Find the value of b.

$$3 - b = 1$$
$$b = 2$$

Find the value of a^2.

$$c^2 = a^2 - b^2$$
$$16 = a^2 - 4$$
$$a^2 = 20$$

Therefore, the equation of the ellipse is $\frac{(x-6)^2}{20} + \frac{(y-3)^2}{4} = 1$.

60. foci at (2, 1) and (2, 13), co-vertex at (5, 7)

SOLUTION:
Since the x-coordinates of the foci are same, the orientation is vertical.

The foci are equidistance from the center. So, the center is at (2, 7).

Therefore, $h = 2$, $k = 7$.

The co-vertex $(h + b, k)$ is (5, 7).

Therefore, $b = 3$.

The value of c is distance between the center and foci.
$c = 6$.
Find the value a^2.

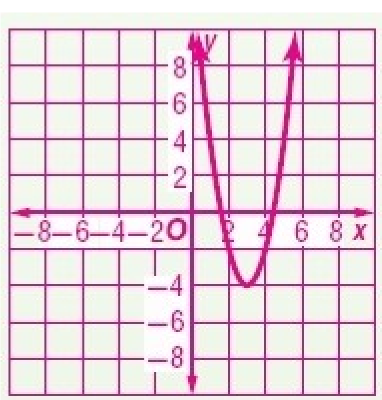
$$c^2 = a^2 - b^2$$
$$6^2 = a^2 - 3^2$$
$$a^2 = 45$$

The equation of the ellipse is $\frac{(x-2)^2}{9} + \frac{(y-7)^2}{45} = 1$.

Graph each function.

61. $y = 2(x - 3)^2 - 4$

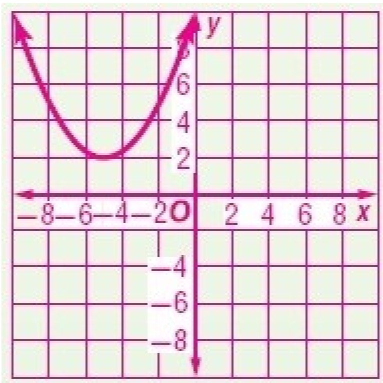
SOLUTION:
The vertex is at (3, –4). The axis of symmetry is at $x = 3$. Because $a = 2 > 1$, the graph is narrower than the graph of $y = x^2$.



62. $y = \frac{1}{3}(x+5)^2 + 2$

SOLUTION:

The vertex is at (–5, 2). The axis of symmetry is at x = –5. Because 0 < a < 1, the graph is compressed vertically.



63. $y = -3(x+6)^2 + 7$

SOLUTION:

The vertex is at (–6, 7). The axis of symmetry is at x = –6. Because a < 0, the graph opens down and is stretched vertically.